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Abstract. The electrovacuum Ernst equations are formulated as a nonlinear σ-model on the symmetric (Kähler) space $SU(1, 2)/S(U(1)\times U(2))$. It is shown, using this formulation, that a generalised Robinson-type identity for the electrovacuum Ernst equations may be derived. A special role played in the derivation of this identity by the hidden symmetry group $SU(1, 2)$ is established. A theorem is proven that the only possible exterior solution for a (pseudo-) stationary, rotating, electrovacuum black hole with non-degenerate event horizon is the Kerr–Newman solution with $m^2-a^2-P^2-Q^2>0$.

1. Introduction

The only possible pure vacuum exterior solution for a stationary, rotating, uncharged black hole with non-degenerate event horizon is the Kerr solution. This uniqueness theorem has been proved by Robinson (1975). Israel (1967) demonstrated that the Schwarzchild solution is the only possible vacuum, stationary, non-rotating (static) and uncharged black hole solution. He was able to generalise this theorem to the electromagnetic case, showing the uniqueness of the Reissner–Nordström black hole solution (Israel 1968). It was still an open question how the result of Robinson would be affected by the presence of an electromagnetic field. There exists a stationary, rotating black hole solution of the source-free Einstein–Maxwell equations—the four-parameter Kerr–Newman solution (Newman *et al* 1965).

The purpose of this paper is to prove uniqueness of the Kerr–Newman black hole solution with $m^2-a^2-P^2-Q^2>0$.

2. The field equations and the black hole boundary conditions

We shall make the simplifying assumption of axisymmetry using the Hawking strong rigidity theorem (Hawking 1972, Ellis and Hawking 1973), according to which a stationary, rotating black hole must be axisymmetric. In the axisymmetric stationary case Carter (1973) has reformulated the Einstein–Maxwell field equations in the manner described by Ernst (1968a, b). The field equations take on the particularly simple form

$$\nabla(\rho\nabla\varepsilon)+\rho X^{-1}(\nabla\varepsilon+2\bar{\psi}\nabla\psi)\nabla\varepsilon=0, \tag{2.1}$$

$$\nabla(\rho\nabla\psi)+\rho X^{-1}(\nabla\varepsilon+2\bar{\psi}\nabla\psi)\nabla\psi=0, \tag{2.2}$$

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where

$$\varepsilon = -X - E^2 - B^2 + \mathrm{i}Y, \qquad \psi = E + \mathrm{i}B, \tag{2.3}$$

and ∇ denotes the covariant derivative with respect to the two-dimensional metric

$$ds^2 = d\lambda^2/(\lambda^2 - c^2) + d\mu^2/(1-\mu^2), \tag{2.4}$$

where the coordinate range of λ and μ is $-1 < \mu < 1$, $c < \lambda < \infty$ and

$$\rho = (\lambda^2 - c^2)^{1/2}(1-\mu^2)^{1/2}. \tag{2.5}$$

The causality requirement implies that X is everywhere positive in the domain of outer communication, apart from the symmetry axis where it is equal to 0.

Carter (1973) has shown that there is a one–one correspondence between the stationary, axisymmetric black hole exterior solution of the Einstein–Maxwell equations and solutions to the equations (2.1) and (2.2) subject to the following boundary conditions.

(i) The asymptotic flatness is ensured by the requirement that E, B, Y and $\lambda^{-2}X$ are well behaved functions of λ^{-1} and μ in the limit $\lambda^{-1} \to 0$ and

$$\begin{aligned} &E = -Q\mu + O(\lambda^{-1}), && B = -P\mu + O(\lambda^{-1}), \\ &Y = 2J\mu(3-\mu^2) + O(\lambda^{-1}), && \lambda^{-2}X = (1-\mu^2)[1 + O(\lambda^{-1})], \end{aligned} \tag{2.6}$$

where J, Q and P are the asymptotically conserved angular momentum, electric and magnetic monopole charges.

(ii) Regularity of the exterior black hole solution on the symmetry axis and the event horizon demand that E, B, X, Y should be well behaved functions of μ and λ there, with

$$\begin{aligned} &E_{,\mu} = O(1), \qquad E_{,\lambda} = O(1-\mu^2), \\ &Y_{,\mu} + 2(EB_{,\mu} - BE_{,\mu}) = O(1-\mu^2), \qquad Y_{,\lambda} = O((1-\mu^2)^2), \end{aligned} \tag{2.7}$$

as $\mu \to \pm 1$ (the symmetry axis) and

$$\begin{aligned} &E_{,\mu} = O(1), && B_{,\mu} = O(1), \\ &E_{,\lambda} = O(1), && B_{,\lambda} = O(1), \\ &Y_{,\lambda} = O(1), && Y_{,\mu} = O(1), \\ &X = O(1), && X^{-1} = O(1), \end{aligned} \tag{2.8}$$

as $\lambda \to c$ (the event horizon).

3. The connection between the Ernst equations and the nonlinear σ-model on the hyperbolic symmetric space SU(1, 2)/S(U(1) × U(2))

The basic point of our demonstration of uniqueness of the Kerr–Newman black hole solution is the generalisation of Robinson's identity to the electromagnetic case (Robinson 1975). The generalisation is the result of inner symmetries of the Ernst equations. In a recent paper of the author (Mazur 1982) it has been shown that the electro-vacuum Ernst equations can be interpreted as the nonlinear σ-model equations on the symmetric (Kähler) space SU(1, 2)/S(U(1)×U(2)); similarly vacuum Ernst equations correspond to the nonlinear σ-model on the SU(1, 1)/U(1) hyperbolic symmetric space (Mazur 1982). Gruszczak (1981) established a connection between the vacuum Ernst equations and the SL(2, R)/SO(2) σ-model. We have worked with the traditional form of the field equations, in terms of the metric components V and W (in the notation of Carter (1973)), which is very convenient for studying the asymptotic properties of space–time, showing that there exists a very simple Lagrangian from which the Ernst equations can be derived. Incidentally, it turned out that this Lagrangian is related in a simple way to that previously obtained by Carter (1973); namely, using $-X$ in place of V we obtain Carter's Lagrangian from ours. The positive definite Lagrangian density written in terms of ε and ψ reads

$$\mathscr{L}=\frac{1}{2}\frac{(\nabla\varepsilon+2\bar\psi\nabla\psi)(\nabla\bar\varepsilon+2\psi\nabla\bar\psi)}{X^2}+2\frac{\nabla\psi\nabla\bar\psi}{X}. \tag{3.1}$$

It is easily seen that under the homographic change of field variables

$$\varepsilon=\frac{\xi-1}{\xi+1},\qquad \psi=\frac{\eta}{\xi+1}, \tag{3.2}$$

the Lagrangian density $\mathscr{L}$ takes the form

$$\mathscr{L}=2(1-\xi\bar\xi-\eta\bar\eta)^{-2}[(1-\eta\bar\eta)\nabla\xi\nabla\bar\xi+(1-\xi\bar\xi)\nabla\eta\nabla\bar\eta+\xi\bar\eta\nabla\eta\nabla\bar\xi+\eta\bar\xi\nabla\xi\nabla\bar\eta], \tag{3.3}$$

which makes the relationship to the nonlinear σ-model on the Kähler symmetric space $D_{1,2}$ transparent. In fact, one can see that the requirement of positivity of X implies

$$\xi\bar\xi+\eta\bar\eta<1, \tag{3.4}$$

and the Lagrangian density $\mathscr{L}$ defines the Bergmann metric $g_{\alpha\bar\beta}$ on the bounded symmetric domain $D_{1,2}$ in C^2 (equation (3.4)) (Kobayashi and Nomizu 1969)

$$\mathrm{d}s^2=g_{\alpha\bar\beta}\,\mathrm{d}z^\alpha\,\mathrm{d}\bar z^\beta=2\Big[\Big(1-\sum_\alpha z^\alpha\bar z^\alpha\Big)\Big(\sum_\alpha \mathrm{d}z^\alpha\,\mathrm{d}\bar z^\alpha\Big)+\Big(\sum_\alpha \bar z^\alpha\,\mathrm{d}z^\alpha\Big)$$
$$\times\Big(\sum_\alpha z^\alpha\,\mathrm{d}\bar z^\alpha\Big)\Big]\Big/\Big(1-\sum_\alpha z^\alpha\bar z^\alpha\Big)^2, \tag{3.5}$$

where $z^1=\xi$ and $z^2=\eta$.

The Lagrangian density (3.3) determines the harmonic map (Misner 1978) between a two-dimensional plane with the metric (2.4) and the Kähler symmetric space $D_{1,2}$ ($D_{1,2}$ = SU(1, 2)/S(U(1)×U(2)). The Kinnersley covariance group SU(1, 2) (Kinnersley 1973) acts transitively (effectively) on $D_{1,2}$.

Eichenherr and Forger (1980) have shown that there exists a natural group-theoretical formulation of two-dimensional nonlinear σ-models of Riemannian symmetric space G/H in terms of a single G-valued field. Let us consider the nonlinear σ-model on the Riemannian symmetric space G/H = SU(p, q)/S(U(p)×U(q)). The symmetric space is a triple (G, H, σ), where G is a connected Lie group, and H is a closed subgroup of G defined by an involutive automorphism σ of G such that $(G_\sigma)_0\subset H\subset G_\sigma$ with G_σ and $(G_\sigma)_0$ being the set of fixed points of σ and its identity component, respectively (Kobayashi and Nomizu 1969).

The Lagrangian density for the σ-model on G/H is given by

$$\mathscr{L}=\tfrac{1}{4}(j_\mu, j^\mu), \tag{3.6}$$

where (,) is a left G-invariant Riemannian metric on G/H and j_μ is the right H-invariant current

$$j_\mu = 2D_\mu g g^{-1}, \tag{3.7}$$

where

$$D_\mu g = g\left(\frac{1-\dot{\sigma}}{2}\right)(g^{-1}\nabla_\mu g), \qquad \dot{\sigma} = \mathrm{d}\sigma|_e \tag{3.8}$$

is the horizontal part of $\nabla_\mu g$.

However, there is a theorem due to Cartan (1930) that a smooth mapping Φ of G/H into G defined by

$$\Phi(g) = g\sigma(g)^{-1}, \tag{3.9}$$

is a diffeomorphism of G/H onto the totally geodesic (closed) submanifold of G. The G-valued field Φ satisfies a constraint

$$\Phi\sigma(\Phi) = I. \tag{3.10}$$

In terms of Φ the current j_μ reads

$$j_\mu = \nabla_\mu \Phi\Phi^{-1}. \tag{3.11}$$

In the case under consideration $\mathrm{G} = \mathrm{SU}(p, q)$ and the involutive automorphism σ is defined by

$$\sigma(g) = \eta g \eta^{-1}, \tag{3.12}$$

where

$$\eta = \begin{pmatrix} -I_p & 0 \\ 0 & I_q \end{pmatrix} \qquad \text{or} \qquad \eta = \mathrm{diag}(\underbrace{-1, \ldots, -1}_{p}, \underbrace{+1, \ldots, +1}_{q}). \tag{3.13}$$

The G-valued field Φ in this case is a Hermitian positive definite matrix

$$\Phi = gg^+, \tag{3.14}$$

because $g \in \mathrm{SU}(p, q)$ implies $\sigma(g)^{-1} = g^+$.

In the case when $p = 1$ Φ can be parametrised in a very simple way, as a consequence of (3.10):

$$\Phi^{\alpha\beta} = \eta^{\alpha\beta} + 2P^{\alpha\beta}, \qquad \alpha = 0, 1, \ldots, q, \tag{3.15}$$

where

$$P^{\alpha\beta} = v^\alpha \bar{v}^\beta \qquad \text{and} \qquad v^\alpha \eta_{\alpha\beta} \bar{v}^\beta \doteq \langle v, v\rangle = -1. \tag{3.16}$$

The last equation (3.16) corresponds to the fact that $\mathrm{SU}(1, q)/\mathrm{S}(\mathrm{U}(1) \times (q))$ is a Hermitian, hyperbolic space and v^α are the Kinnersley coordinates on this space (Kinnersley 1973). Introducing other coordinates on $\mathrm{SU}(1, q)/\mathrm{S}(\mathrm{U}(1) \times \mathrm{U}(q))$ defined by

$$\xi^\alpha = v^\alpha / v_0, \qquad \alpha = 1, \ldots, q, \tag{3.17}$$

one sees easily that the Lagrangian density for a nonlinear σ-model on this symmetric space,

$$\mathscr{L} = \tfrac{1}{4}\mathrm{Tr}(j_\mu j^\mu), \tag{3.18}$$

is equivalent to the Lagrangian density for the Ernst electrovacuum equations (3.3) when $q=2$ and

$$\xi^1=\xi, \qquad \xi^2=\eta. \tag{3.19}$$

Similarly, the case $q=1$ corresponds to pure vacuum Ernst equations.

The field equations derived from the Lagrangian density (3.18) have the form

$$\nabla_\mu(\rho j^\mu)=0, \tag{3.20}$$

and are equivalent to the Noether conservation law implied by the invariance of the action integral under global left G(= SU(p, q))-translations.

4. The identity and the proof of uniqueness of the Kerr–Newman black hole solution

We shall demonstrate that the Robinson identity (Robinson 1975) is a consequence of the inner (hidden) symmetry of Ernst's equations. In the formulation of the Ernst equations presented above, the identity appears naturally in its G-invariant form ($G=SU(1,q)$, $q=1,2$).

Let us consider two fields Φ_1 and Φ_2, not necessarily solutions of the field equations (3.20), and define a field Φ by

$$\Phi=\Phi_1\Phi_2^{-1}. \tag{4.1}$$

One can see from (3.14) that Φ has the form

$$\Phi=g_1Mg_1^{-1}, \tag{4.2}$$

where

$$M=g^+g \qquad \text{and} \qquad g=g_2^{-1}g_1. \tag{4.3}$$

Φ transforms under the left G-translations, $g_1\to ug_1$ and $g_2\to ug_2$ as follows:

$$\Phi^1=u\Phi u^{-1}, \qquad u\in G. \tag{4.4}$$

Now, having defined a field Φ, we consider, using (3.11) and (4.1), the following identities:

$$\nabla_\mu\Phi=j_\mu^{(1)}\Phi-\Phi j_\mu^{(2)}, \tag{4.5}$$

where

$$j_\mu^{(i)}=\nabla_\mu\Phi^{(i)}\Phi^{(i)-1}, \qquad i=1,2, \tag{4.6}$$

and

$$\nabla_\mu(\rho\nabla^\mu\Phi)=\nabla_\mu(\rho j^{(1)\mu})\Phi-\Phi\nabla_\mu(\rho j^{(2)\mu})+\rho(j_\mu^{(1)}j^{(1)\mu}\Phi+\Phi j_\mu^{(2)}j^{(2)\mu}-2j_\mu^{(1)}\Phi j^{(2)\mu}). \tag{4.7}$$

The form of the identity invariant under the global left G-translations can be obtained by taking the trace of (4.7). Then, we have the final G-invariant form of the identity

$$\begin{aligned}\mathrm{Tr}\{\Phi[\nabla_\mu(\rho j^{(2)\mu})-\nabla_\mu(\rho j^{(1)\mu})]\}+\mathrm{Tr}\,\nabla_\mu(\rho\nabla^\mu\Phi)\\=\rho\,\mathrm{Tr}\{\Phi[j_\mu^{(1)}j^{(1)\mu}+j_\mu^{(2)}j^{(2)\mu}-2j_\mu^{(2)}j^{(1)\mu}]\}.\end{aligned} \tag{4.8}$$

The identity (4.8) is the generalised Robinson identity for the nonlinear σ-model on the $SU(p,q)/S(U(p)\times U(q))$ symmetric space. For the case when $p=1$ and $q=1,2$

we have the pure vacuum Robinson identity and its generalisation to the electrovacuum case. The main advantage of this form of the identity is its independence of the particular parametrisation of SU(1, q)/S(U(1) × U(q)) and the manifest invariance under the hidden symmetry group of Ernst's equations. Taking the Ernst parametrisation of SU(1, q)/S(U(1) × U(q)), we have

$$\Phi^{\alpha\beta} = \eta^{\alpha\beta} - 2\xi^{\alpha}\bar{\xi}^{\beta}/\langle\xi, \xi\rangle, \tag{4.9}$$

where

$$\langle\xi, \xi\rangle = -1 + \sum_{\alpha} \xi^{\alpha}\bar{\xi}^{\alpha}, \qquad \alpha = 1, \ldots, q. \tag{4.10}$$

In particular, we have for the vacuum and electrovacuum cases

$$\Phi = (1 - \xi\bar{\xi})^{-1}\begin{pmatrix} 1 + \xi\bar{\xi} & 2\bar{\xi} \\ 2\xi & 1 + \xi\bar{\xi} \end{pmatrix}, \tag{4.11}$$

$$\Phi = (1 - \xi\bar{\xi} - \eta\bar{\eta})^{-1}\begin{pmatrix} 1 + \xi\bar{\xi} + \eta\bar{\eta} & 2\bar{\xi} & 2\bar{\eta} \\ 2\xi & 1 + \xi\bar{\xi} - \eta\bar{\eta} & 2\xi\bar{\eta} \\ 2\eta & 2\eta\bar{\xi} & 1 + \eta\bar{\eta} - \xi\bar{\xi} \end{pmatrix}, \tag{4.12}$$

respectively.

In order to draw conclusions about the uniqueness of solutions to the Ernst equations we need only the form of $\mathrm{Tr}(\Phi_1\Phi_2^{-1})$ in terms of Ernst's potentials ε_1, ψ_1, ε_2, and ψ_2 (for the electrovacuum case) and the proof of positivity of the right-hand side of the identity (4.8).

From (3.2), (4.11) and (4.12) we have

$$\mathrm{Tr}\,\Phi = \mathrm{Tr}(\Phi_1\Phi_2^{-1}) = 2 + X_1^{-1}X_2^{-1}[(X_1 - X_2)^2 + (Y_1 - Y_2)^2], \tag{4.13}$$

$$\begin{aligned}\mathrm{Tr}\,\Phi = 3 + X_1^{-1}X_2^{-1}\{&(X_1 - X_2)^2 + 2(X_1 + X_2)[(E_1 - E_2)^2 + (B_1 - B_2)^2] \\ &+ [(E_1 - E_2)^2 + (B_1 - B_2)^2]^2 + [Y_1 - Y_2 + E_2B_1 - E_1B_2]^2\},\end{aligned} \tag{4.14}$$

for the vacuum and electrovacuum cases, respectively.

It can easily be seen from (4.13) that the total divergence term of the identity (4.8) corresponds to that of Robinson (1975). One should note that the positivity of a Hermitian matrix M in (4.3) implies that $\mathrm{Tr}\,\Phi$ is real and

$$\mathrm{Tr}\,\Phi = \mathrm{Tr}\,M \geqslant 1 + q, \tag{4.15}$$

where the equality holds when $g \in \mathrm{H} = \mathrm{S}(\mathrm{U}(1) \times \mathrm{U}(q))$ and therefore $\Phi_1 = \Phi_2$, $\Phi = I$. It is in agreement with (4.13) and (4.14). The right-hand side of the identity (4.8) is real and positive. This can be seen as follows. From (3.11) and (3.14) we have

$$j_{\mu}^{(i)} = g_i k_{\mu}^{(i)} g_i^{-1}, \qquad i = 1, 2, \tag{4.16}$$

where

$$k_{\mu}^{(i)} = \nabla_{\mu} g_i^{+}(g_i^{+})^{-1} + g_i^{-1}\nabla_{\mu} g_i, \tag{4.17}$$

and

$$k_{\mu}^{(i)+} = k_{\mu}^{(i)}. \tag{4.18}$$

Using (3.11), (4.2), (4.3), (4.16) and (4.18) we have

$$R = \rho\,\mathrm{Tr}\{g k_{\mu}^{(1)} k^{(1)\mu} g^{+} + g^{+} k_{\mu}^{(2)} k^{(2)\mu} g - 2k_{\mu}^{(2)} g k^{(1)\mu} g^{+}\}, \tag{4.19}$$

where R denotes the right-hand side of the identity (4.8).

Let us define the matrices

$$P^{(1)} = gk^{(1)}, \qquad P^{(2)} = k^{(2)}g. \tag{4.20}$$

Using the hermiticity of $k^{(i)}$ and (4.20) we have

$$R = \rho\ \mathrm{Tr}(P^{(1)}_{\mu}P^{(1)\mu+} + P^{(2)}_{\mu}P^{(2)\mu+} - 2P^{(2)}_{\mu}P^{(1)\mu+}). \tag{4.21}$$

R is real if the third term in (4.21) is real. One can see from (4.18) and (4.20) that the third term in (4.21) is real and

$$\mathrm{Tr}(P^{(2)}_{\mu}P^{(1)\mu+}) = \mathrm{Tr}(P^{(1)}_{\mu}P^{(2)\mu+}). \tag{4.22}$$

Therefore, we have

$$R = \rho\ \mathrm{Tr}[(P^{(1)} - P^{(2)})_{\mu}(P^{(1)+} - P^{(2)+})^{\mu}] = \rho\ \mathrm{Tr}\{m_{\mu}m^{\mu+}\} \geqslant 0, \tag{4.23}$$

where

$$m = P^{(1)} - P^{(2)} = g_2^{-1}(j^{(1)} - j^{(2)})g_1. \tag{4.24}$$

If the right-hand side R of the identity (4.8) vanishes then m is equal to zero. From (3.7) and (3.8) it follows that m can be written in the form

$$m = Dg = g\left(\frac{1-\dot{\sigma}}{2}\right)(g^{-1}\nabla g). \tag{4.25}$$

Equation (4.25) implies that there are two possibilities if the right-hand side R of the identity vanishes:

(i) $Dg = 0$ if g is a local function and $g \in \mathrm{H}$. This leads to the conclusion that $\Phi_1 = \Phi_2$ independently of the boundary conditions on Φ_1 and Φ_2.

(ii) $Dg = 0$ if g is a constant matrix and $g \in \mathrm{G}(=\mathrm{SU}(1, q))$.

Proof of uniqueness of the Kerr-Newman black hole solution

Let (X_1, Y_1, E_1, B_1) and (X_2, Y_2, E_2, B_2) (or Φ_1 and Φ_2) be two black hole solutions of the Einstein–Maxwell equations with a regular domain of outer communication satisfying Carter's boundary conditions (2.6), (2.7) and (2.8). Suppose that (X_1, Y_1, E_1, B_1) corresponds to the Kerr–Newman solution with $m^2 - a^2 - P^2 - Q^2 > 0$. The boundary integral which appears on the left-hand side of the identity (4.8) integrated over the two-dimensional space, after the application of Stokes' theorem, can be shown to vanish if the boundary conditions (2.6), (2.7) and (2.8) are satisfied. This implies that the right-hand side R of the identity (4.8) must vanish. However, R vanishes if (ii) (the second possibility) is satisfied. This leads to a simple relation on the solutions Φ_1 and Φ_2

$$\mathrm{Tr}\,\Phi = \mathrm{Tr}(g^{+}g) = c, \tag{4.26}$$

where c is a constant.

The application of the boundary conditions to the equations (4.14) and (4.26) leads to

$$c = \mathrm{Tr}\,\Phi = 3, \tag{4.27}$$

and hence

$$\Phi_1 = \Phi_2. \tag{4.28}$$

This shows the uniqueness of the Kerr–Newman black hole solution.

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